



# Pergroup Precoding For MIMO With LDPC Coding Using QAM Modulation

<sup>1</sup>S.Iniya, M.E student, Priyadarshini Engineering College, Vaniyambadi

E-Mail id: iniya412@gmail.com

**Abstract**—In this paper, the problem of designing a linear precoder for Multiple-Input Multiple-Output (MIMO) systems employing Low-Density Parity-Check (LDPC) codes is addressed under the constraint of minimizing the dependence between the system's receiving branches, thus reducing the relevant transmitter and receiver complexities. Our approach constitutes an interesting generalization of Bit-Interleaved Coded Modulation with Multiple Beamforming (BICMB) which has shown many benefits in MIMO systems. We start with a Pareto optimal surface modeling of the system and show the difficulty involved in the corresponding optimization problem. We then propose an alternative, practical technique, called Per-Group Precoding (PGP) with QAM modulation, which groups together multiple input symbol streams and corresponding receiving branches in the "virtual" channel domain (after singular value decomposition of the original MIMO channel), and thus results in independent transmitting/receiving streams between groups. We show with numerical results that PGP offers almost optimal performance, albeit with significant reduction both in the precoder optimization and LDPC EXIT chart based decoding complexities.

**Index Terms**—Per-group processing, maximal diversity precoder, near-capacity achieving channel coding, pareto optimal precoder, receiver independence factor.

## I. INTRODUCTION

**THE** concept of Multiple-Input Multiple-Output (MIMO) still represents a prevailing research direction in wireless communications due to its ever increasing capability to offer higher rate, more efficient communications, as measured by spectral utilization, and under low transmitting or receiving power. Within MIMO research, BICMB [1]–[3] has shown great potential for practical application, due to its excellent diversity gains and its simplicity. For

example, BICMB in conjunction with convolutional coding offers maximum diversity and maximum spatial multiplexing simultaneously [1], thus it represents an optimal technique for this type of Forward Error Correction (FEC). In addition, there are many past works available which investigated with success linear precoding through exploitation of a unitary precoding matrix, mainly from a diversity maximization point of view [4], [5]. On the other hand, LDPC coding is the currently prevailing, near-capacity achieving error-correction technique that operates based on input to output mutual information and extrinsic information transfer (EXIT) charts [6], [7]. The problem of designing an optimal linear precoder toward maximizing the mutual information between the input and output was first considered in [8], [9] where the first optimal power allocation strategies are presented (e.g., Mercury Waterfilling (MWF)), together with general equations for the optimal precoder design. In addition, [10] also considered precoders for mutual information maximization and showed that the left eigenvectors of the optimal precoder can be set equal to the right eigenvectors of the channel. Finally, in [11], a mutual information maximizing precoder for a parallel layer MIMO detection system is presented reducing the performance gap between maximum likelihood and parallel layer detection. Recently, globally optimal linear precoding techniques were presented [12], [13] for finite alphabet inputs, capable of achieving mutual information rates much higher than the previously presented MWF [8] techniques, by introducing input symbol correlation through a unitary input transformation matrix in conjunction with channel weight adjustment (power allocation). These mutual information maximizing globally optimal precoders are more appropriate for LDPC codes which are very popular currently, than e.g., Maximal Diversity Precoders (MDP) [12]. However, the gains presented

in [12], [14], [15] are achieved at the expense of significantly increased system complexity, even for small modulation constellation size  $M$  (e.g.,  $M = 2, 4$ ). In addition, the interesting design of [12] requires a significant computational complexity increase at the receiver, even in its simplified implementation for  $M$ -ary Phase Shift Keying (MPSK) systems [14], due to e.g., the dependence present between receiving branches. This increase could be prohibitive if the receiver is the mobile destination, or if the number of receiving branches is high. In this paper, we propose linear precoding techniques which offer high mutual information between input and output in a MIMO system with Quadrature Amplitude Modulation (QAM) and also offer semi-independence among the receiving branches, thus highly simplifying the Maximum A Posteriori probability (MAP) detector operation, and hence significantly reducing the receiver complexity. First, we investigate the theoretical aspects of jointly maximizing the mutual information rate and the independence of the receiving branches. This is done within a Pareto optimal surface context. We show that this problem becomes very involved due to the non-concavity of the functions involved and thus more practical

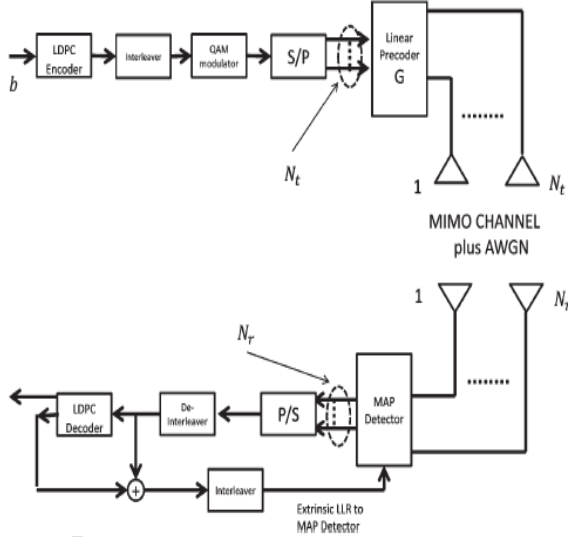


Fig. 1. Original MIMO System model at baseband with Linear Precoder  $G$ ,  $N_t$  transmitting antennas,  $N_r$  receiving antennas, and LDPC encoder and decoder. A generic coding rate  $R$  is assumed for the LDPC encoder. A similar system can be built for the equivalent channel presented.

solutions should be sought. We then proceed to propose a new, interesting technique that groups together “similar” small numbers of multiple streams of input data and receiving branches and then it applies optimized precoding on each group. We

carefully look at the group selection strategy, and we show that the best selection is based on selecting input and output groups based on maximum separation of their singular values. The proposed technique is named Per Group Precoding (PGP). PGP offers very good performance with significantly reduced complexity both at the precoder design and receiver levels, due to the independence among different groups and it can be successfully applied even to QAM constellations with  $M \geq 16$ .

## II. LINEAR PRECODER OPTIMIZATION WITH REDUCED COMPLEXITY

The  $N_t$  transmit antenna,  $N_r$  receive antenna MIMO model (Fig. 1) is described by the following equation

$$\mathbf{y} = \mathbf{H}\mathbf{G}\mathbf{x} + \mathbf{n},$$

where  $\mathbf{y}$  is the  $N_r \times 1$  received vector,  $\mathbf{H}$  is the  $N_r \times N_t$  MIMO channel matrix comprising independent complex Gaussian components of mean zero and variance one and assumed quasi-static [1],  $\mathbf{G}$  is the precoder matrix of size  $N_t \times N_t$ ,  $\mathbf{x}$  is the  $N_t \times 1$  data vector with independent, identically distributed components of (normalized) power one (thus with covariance matrix  $\mathbf{K}_X = \mathbf{I}_{N_t}$ ), each of which is in the QAM constellation, and  $\mathbf{n}$  represents the circularly symmetric complex Additive White Gaussian Noise (AWGN) of size  $N_r \times 1$ , with mean zero and covariance matrix  $\mathbf{K}_n = \sigma^2 n \mathbf{I}_{N_r}$ , where  $\mathbf{I}_{N_r}$  is the  $N_r \times N_r$  identity matrix, and  $\sigma^2 n = 1 / \text{SNR}$ ,  $\text{SNR}$  being the (coded) symbol signal-to-noise ratio. The precoding matrix  $\mathbf{G}$  needs to satisfy the following power constraint

$$\text{tr}(\mathbf{G}\mathbf{G}^H) = N_t$$

where  $\text{tr}(\mathbf{A})$ ,  $\mathbf{A}^H$  denote the trace, and the Hermitian transpose of matrix  $\mathbf{A}$ , respectively.

An equivalent model, assuming without loss of generality  $N_t v = N_r v = N_t$  in the virtual domain (after singular value decomposition (SVD)), where  $N_t v, N_r v$  represent the transmitting and receiving antennas in the virtual domain, respectively, called herein the “virtual” channel can be easily built based on [12] as follows

$$\mathbf{y} = \mathbf{\Sigma}\mathbf{H}\mathbf{\Sigma}\mathbf{G}\mathbf{V}\mathbf{h}\mathbf{G}\mathbf{x} + \mathbf{n}$$

where  $\mathbf{\Sigma}\mathbf{H}$  and  $\mathbf{\Sigma}\mathbf{G}$  are diagonal matrices containing *non-zero* singular values of  $\mathbf{H}$ ,  $\mathbf{G}$ , respectively, padded with zeroes if necessary for dimension consistency, and  $\mathbf{V}\mathbf{G}$  is the matrix of the right singular vectors of  $\mathbf{G}$ . Thus, all matrices in (3) are of size  $N_t \times N_t$ , while the vectors are of size  $N_t \times 1$ . When LDPC coding with sufficient blocklength (see below) is employed in this MIMO system, the overall



utilization in  $b/s/Hz$  is determined by the mutual information between the transmitting branches  $x$  and the receiving ones,  $y$  [6], [7]. It is shown [12] that the mutual information between  $x$  and  $y$ ,  $I(x;y)$ , is only a function of  $W = VG\Sigma^2 H\Sigma^2 GVh G$ , an important property toward mutual information maximization. The optimal precoder  $G$  is found by solving

$$\begin{aligned} & \text{maximize } G I(x;y) \\ & \text{subject to } \text{tr}(GGh) = Nt \end{aligned}$$

In (4), the constraint present is due to the total (normalized) average MIMO input power which needs to be kept equal to  $Nt$ . The average MIMO input power is given as  $PMIMO = E(\text{tr}(GxxhGh)) = \text{tr}(GGh)$ , due to the assumptions made on  $x$ . The solution to (4) results in exponential complexity at both transmitter and receiver, as shown in Section II. When an appropriate LDPC code of sufficient blocklength  $Nb$  is employed in the described MIMO system, in conjunction with Gray coding and interleaving employed at the transmitter, followed by MAP detector at the receiver, the system offers very low bit error rate (BER) (e.g.,  $BER < 10^{-4}$ ) [6], [7], provided that the coding rate of the LDPC code satisfies the condition

$$R < I(x,y)Nt \log_2(M)$$

This is due to the fact that LDPC codes are near-capacity achieving codes. For example, based on the published results in [12], a blocklength  $Nb \geq 2400$  would suffice toward meeting (5) closely for a  $2 \times 2$  MIMO system. Thus, designing precoders for high input-output mutual information is more appropriate for LDPC systems than other type of precoders, e.g., MDP. Based on this fact, we focus on this type of precoder designs without special attention on the LDPC code design details. Due to SVD, the original  $Nr \times Nt$  system described by (1) is equivalent to a size  $Nt \times Nt$  virtual MIMO system. Other types of near-capacity achieving channel coding, e.g., Turbo coding could also be employed in our MIMO precoding schemes, as well. However, as LDPC codes represent one form of the currently prevailing channel coding techniques, we decided to focus on this type of channel coding herein.

### A. Pareto Surface Precoder Optimization

In this paper, we are interested in precoders which are capable of maximizing jointly  $I(x;y)$  and a measure of independence between the elements of  $y$  in (3). Our rationale is that these types of precoders lead to simpler designs with reduced complexity both at the transmitter and the receiver. To see this, we use the fact that higher values of the degree of independence between the elements of  $y$  are

facilitated by independent partitions of the MIMO channel input-output space into independent groups. An input-output MIMO Independent Group Partition (IGP) with  $N_g$  groups,  $P$ , is defined as follows:  $P = \{S_1, \dots, S_{N_g}\}$ , where each group in the partition,  $S_i$  with  $1 \leq i \leq N_g$ , comprises  $N_{gi}$  unique, non-intersecting input-output vector pairs,  $(x_{Si}, y_{Si})$ , which satisfy  $\cup_{i=1}^{N_g} (x_{Si}) = \cup_{i=1}^{N_g} (y_{Si}) = \{1, 2, \dots, Nt\}$  and  $N(x_{Si}) \cap_{j=iN} (x_{Sj}) = N(y_{Si}) \cap_{j=iN} (y_{Sj}) = \emptyset$ , where  $N(\cdot)$  represents the index set (nodes) present in the argument set (within the parenthesis). In addition, IGP requires that each input node,  $n$ , in a partition set  $S_i$  is connected only to the corresponding outputs in  $S_i$ , in other words, the elements of  $VG$  corresponding to all other outputs are set to zero, i.e.,  $VG(n, j) = 0$ , for output nodes  $j$  with  $j \notin N(y_{Si})$ , and where  $VG(n, j)$  represents the  $i$ th row,  $j$ th column entry of matrix  $VG$  of (3). All the IGP schemes considered herein use square group structure, i.e., with same number of input and output elements in each group. In other words,  $|N(x_{Si})| = |N(y_{Si})| = N_{vi}$ , with  $\sum_{i=1}^{N_g} N_{vi} = Nt$ . Thus, the IGP structure clearly defines  $N_g$  independent MIMO input-output groups, i.e.,  $N_g$  independent, smaller dimension MIMO systems. In the sequel we will also need the notion of an Output only Independent Group Partition (OIGP). Let  $p(\cdot)$  be the pdf of the quantity within the parenthesis, and let  $\text{Pr}(\cdot)$  be the probability of the quantity within the parenthesis. An OIGP MIMO system is defined as a partition of the output vector  $y$  into  $N_g$  independent groups, i.e., with  $p(y) = \prod_{l=1}^{N_g} p(y_{Sl})$ . An interesting fact is that an AWGN MIMO system as defined by (3) is equivalent to an IGP with the same number of groups,  $N_g$ , as we show below. Now we present two lemmas that are essential to the overall paper understanding, with their proofs presented in the Appendices.

**Lemma 2.1:** Any MIMO OIGP with  $N_g$  groups is equivalent to a MIMO IGP with the same number of groups. Based on this lemma, we see that seeking for higher degree of independence in the output of the MIMO system is equivalent to searching for an IGP. Now, let  $S(l)$  ( $1 \leq l \leq Nt$ ), be the mapping from the  $l$ th input  $x_l$  to its corresponding group, and let  $|S(l)|$  be the number of elements in this group.

**Lemma 2.2:** Any MIMO OIGP with  $N_g$  groups results in an equivalent expression of the symbol MAP probability,  $\text{Pr}(x_i/y)$ , as follows

$$\text{Pr}(x_i/y) = \text{Pr}(x_i/y_{S(i)})$$

where we have used  $\text{Pr}(\cdot)$  since  $x_i$  takes values in a discrete set.



Due to this lemma, the MAP detector with OIGP needs to only look at the MIMO system defined by  $S(i)$  to calculate the MAP probability, thus simplifying the overall MAP detector complexity as only  $N_{vi} < N_t$  input and output nodes are present in the MAP detector of  $x_i$ . Then each group can be separately precoded and decoded at the receiver thus simplifying the overall system complexity. This reduction in the receiver complexity is evaluated in more detail in Section III. A widely accepted measure of independence among the elements of a random vector is the difference between the joint entropy and the sum of all marginal entropies, which herein is called Receiver Independence Factor (RIF)

$$RIF = H(\mathbf{y}) - H(\mathbf{y}) = H(\mathbf{y}) - N_r \sum_{i=1}^{N_g} H(y_i), \quad (6)$$

where  $H(\cdot)$  represents the entropy of the argument in the parenthesis,  $y_i$  is the  $i$ th receiving branch, and we defined  $H(y) = \sum_{r=1}^{N_r} H(y_r)$ . Thus, to exploit a high RIF jointly with a high mutual information, one should aim at jointly maximizing over  $G$  the vector  $[H(\mathbf{y})RIF]^T$  over the (proper) cone  $\mathbb{R}^2_+$ , satisfying the power constraint  $\text{tr}(GG^H) = N_t$ , resulting in a multicriterion problem [16]. A scalarization of this problem simplifies the problem complexity dramatically and results in a well-accepted method to recover optimal points of the original problem, where  $f_0(x)$  is the objective function. In addition, scalarization and Pareto optimal point finding is the natural way to study multicriterion problems such as the one we are dealing with here. For problems over  $\mathbb{R}^2_+$ , i.e., with  $f_0(x) = [f_01(x) f_02(x)]^T$ , scalarization leads to maximizing over  $x$  the Pareto surface scalar function  $f_01(x) + \lambda_0 f_02(x)$  with  $\lambda_0 \geq 0$ .

### B. A Different Optimization Problem

For any  $N_g$  group partition of the “virtual” receiving branches,  $y$  in the equivalent model (3), i.e.,  $N(y_{Si}) \cap_{j=1}^{N_g} (y_{Sj}) = \emptyset$  and  $\cup_{i=1}^{N_g} (y_{Si}) = \{1, 2, \dots, N_t\}$ , (15) where  $N(\cdot)$  represents the index set (nodes) present in the argument set (within the parenthesis), the following inequality is true for fixed  $V_G, \Sigma_G$ :  $H(\mathbf{y}) \leq N_g \sum_{i=1}^{N_g} H(y_{Si}) \leq N_r \sum_{i=1}^{N_g} H(y_i)$ . This is a very fundamental result in information theory [20] and equality holds if and only if the group outputs are independent. Thus, a general partitioning in larger groups brings the MIMO output entropy closer to the partition one (note that this applies to the MIMO input-output mutual information too, due to its irrelevant constant difference from the entropy value). Consider the following optimization problem: maximize  $V_G, \Sigma_G H(\mathbf{y}) - N_g \sum_{i=1}^{N_g} H(y_{Si})$  subject to  $N(y_{Si}) \cap_{j=1}^{N_g} (y_{Sj}) = \emptyset$  and  $\cup_{i=1}^{N_g} (y_{Si}) = \{1, 2, \dots, N_t\}$ .

(17) The solution of this problem leads to independence between the  $N_g$  output groups, i.e.,  $H(\mathbf{y}) = \sum_{i=1}^{N_g} H(y_{Si})$  (e.g. [20]), creating an OIGP, as explained above. Based on Lemma 1, an OIGP for this type of AWGN MIMO channel is equivalent to an IGP, thus inputs need to be also partitioned into the same number of groups, wherein each input group is only connected to one output group only. To take advantage of this, consider the following problem: maximize  $V_G, \Sigma_G H(\mathbf{y})$  subject to  $H(\mathbf{y}) = N_g \sum_{i=1}^{N_g} H(y_{Si})$  and  $\text{tr}(\sum_{i=1}^{N_g} G_i) = N_t$ . This way, the previous result can be easily utilized to exploit this type of inter-group independence. This is the generalized PGP problem (G-PGP). A simpler version is obtained if we further specialize the power constraint in (18) to  $\text{tr}(\sum_{i=1}^{N_g} G_i) = N_t i$ , for  $i = 1, 2, \dots, N_g$ . The corresponding solution is called Per Group Precoding (PGP) and it is found as follows: For a particular variable selection method, let  $x_{si}, y_{si}$  be the data variables and the receiving vector variables in the  $i$ th selection subset (group), respectively. Let us denote by  $N_{ti}, N_{ri}, N_g$ , the numbers of spatially multiplexed data streams, spatially multiplexed receiving antennas per group, and of PGP groups, respectively, then,  $N_t = \sum_{i=1}^{N_g} N_{ti}$  and  $N_r = \sum_{i=1}^{N_g} N_{ri}$ . PGP solves the following  $N_g$  optimization sub-problems, one for each  $I$  (group) ( $i = 1, 2, \dots, N_g$ ): maximize  $W_{si} I(x_{si}, y_{si})$  subject to  $W_{hs} i = W_{si}$  and  $\text{tr}(\sum_{i=1}^{N_g} G_i) = N_{ti}$ . Theorem 2.8: The PGP solution is in the feasible region of the original problem (9). This is due to the fact that the solution offered by PGP in the equivalent model.

### C. Computational Complexity Evaluation and Comparison

The original linear precoder (global) optimization problem as described by (9) is solved in [12]. Assuming  $N_r = N_t$ , as it is the case for the presented results herein, we can perform a basic calculation of the complexity involved at the transmitter and receiver sites of a MIMO system employing linear precoding techniques to increase the input-output mutual information. We are evaluating the computational complexity order  $O$  for the different transmitter and receiver processing stages employed.  $O$  represents the total number of calculations, including the number of real additions, and multiplications required for a task, expressed as a constant times the dominating calculation factor involved in the task. The computational complexity order is then this calculation dominating factor of the task.

1) **Evaluation of Transmitter Complexity:** Table I shows the complexity order for the different precoder

stages at the transmitter. These are the gradient evaluations (GE), as required by the two backtracking line search algorithms, the two required evaluations of the objective function  $I$  (OE) per backtracking line search iteration, and the calculation of  $d\Theta$  of the differential of the unitary matrix  $\Theta$  (UE). We see that the transmitter computational complexity order of a global optimal precoder is  $M2Nt$ .

2) **Evaluation of Receiver Complexity:** The major ramification introduced at the receiver due to transmitter precoding is the one regarding the MAP detector [6], [7] which evaluates the channel bit log-likelihood-ratios (LLR). Table II shows the corresponding complexity which is  $O(M2Nt \log_2(M))$ .

3) **Comparison of PGP to Global Optimization Complexity:** Concerning the comparison of PGP complexity versus the globally optimal precoder one for the precoder and the MAP detector, when receiver branches are partitioned in independent groups (e.g., high values of RIF), each group represents exponentially smaller complexity both at the precoder determination and at the MAP detector, due to intergroup independence and smaller dimensions per group. To see how PGP reduces the MAP detector complexity, consider PGP in a MIMO system with  $N_g$  IGP groups. Let us focus on calculating the probability  $\Pr(x_i/y) = \Pr(x_i/yS(i))$ , for input symbol  $x_i$  in the system. Since PGP relies on an IGP to apply, let us use a corresponding IGP,  $P = \{S_1, \dots, S_{N_g}\}$ , we have  $N_g$  groups of nodes at both input and output domain. Each group in the partition,  $S_i$  with  $1 \leq i \leq N_g$ , comprises  $N_{gi}$  unique, non-intersecting input-output vector pairs,  $(x_{Si}, y_{Si})$ , as explained earlier, with  $\sum_{i=1}^{N_g} N_{vi} = Nt$ . Let  $nin(i, j), no(i, j)$  be the mapping from the partition groups to input and output indices, respectively. In other words,  $nin(i, j)$  represents the input index (taking values in  $\{1, \dots, Nt\}$ ) of the  $j$ th ( $1 \leq j \leq N_{vi}$ ) input of the  $i$ th group ( $1 \leq i \leq N_g$ ), and similarly for  $no(i, j)$ . Also, let  $S(l) (1 \leq l \leq Nt)$ , be the mapping from the  $l$ th input  $x_l$  to its corresponding group. Finally, let  $xS(i) = \{x_{nin(i,1)}, \dots, x_i, \dots, x_{nin(i,S(i))}\}$

### III. NUMERICAL RESULTS

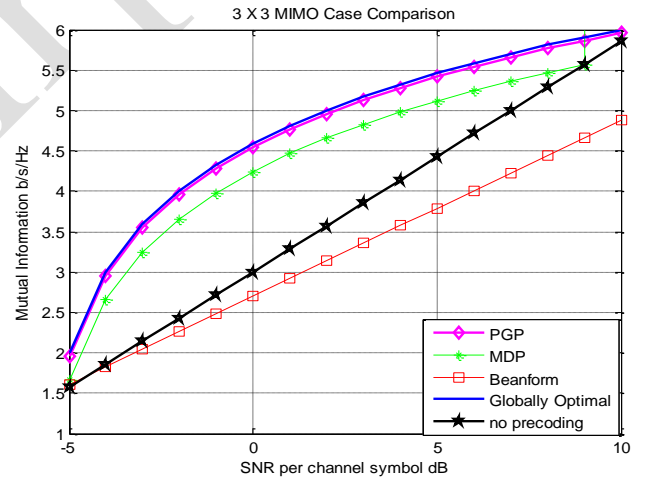
The results presented herein employ PGP as described above, as well as globally optimal precoders for comparison. Since PGP performs a number of  $N_g$  globally optimal precoder determinations, albeit of smaller size, it suffices to describe the globally optimal recoder implementation. The globally optimal precoder implementation methodology is

performed by employing two backtracking line searches, one for  $W$ , and another one for  $\Sigma^2 G$ , at each iteration, in a fashion similar to [12], but introducing some improvements. Similarly to [12], we follow a block coordinate gradient ascent maximization method to find the solution to the optimization problem described in (9), employing the virtual model of (3). It is proven in [12] that  $I(x;y)$  is a concave function over  $W$  and  $\Sigma^2 G$ . It thus becomes efficient to employ two different gradient ascent methods, one for  $W$ , and another one for  $\Sigma^2 G$ .

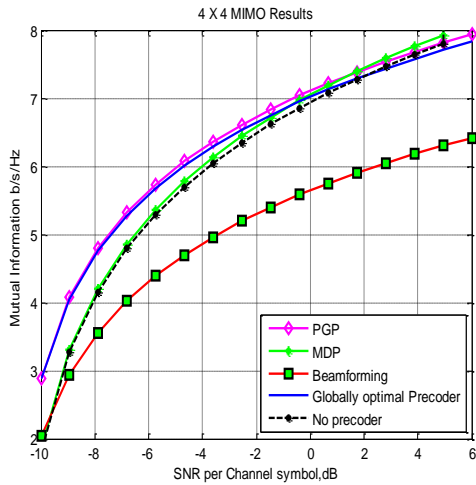
All the results consider Quadrature Phase Shift Keying (QPSK) modulation on narrowband independently fading channels. For a  $3 \times 3$  MIMO system with

$$\mathbf{H} = \begin{bmatrix} 1 & 0.5J & 0.3 \\ -0.5J & 1.5 & -0.1J \\ 0.3 & 0.1J & 0.5 \end{bmatrix}$$

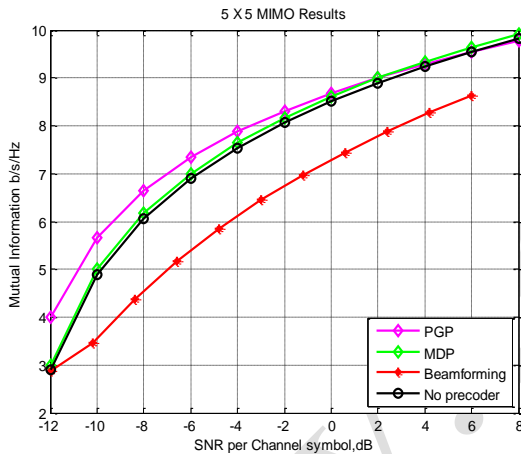
The output of  $3 \times 3$  MIMO system is,



The output of 4x4 MIMO matrix is,



The output of 5x5 MIMO matrix is,



#### IV. CONCLUSION

In this paper, the problem of designing a linear precoder for MIMO systems employing LDPC codes is addressed under the constraint of minimizing the dependence between the system's receiving branches, thus reducing the relevant transmitter and receiver complexities. This approach sees the overall precoding problem in an LDPC-coded system from a brand new angle allowing for a more practical deployment of higher dimension MIMO systems and higher QAM constellation sizes with very good performance over a wide SNR range. We show that a Pareto surface optimization leads to an intractable problem, in general. We then target a generalization of BICMB (which is the only tractable solution to the  $\lambda = 1$  Pareto problem) and show that this offers a solution to a very meaningful precoding optimization problem

that allows for inter-group independence between different transmitting/receiving antenna pairs in the virtual channel domain. We call the new precoding solution PGP and we show, based on numerical results, that PGP offers indeed excellent performance, while its computational complexity is significantly reduced compared to the original solution. For the results presented, PGP is shown to attain mutual information very close to the globally optimal precoder and higher than MDP by 0.7–0.8 b/s/Hz in medium SNR ranges. Thus, based on presented evidence, PGP is a very good candidate for almost optimal precoding performance in LDPC coded systems with relatively low system complexity at both the transmitter and receiver. Our future work will look at generalizing the presented approach to higher size modulation constellations, e.g., QAM with  $M \geq 16$ .

#### REFERENCES

- [1] E. Akay, E. Sengul, and E. Ayanoglu, "Bit interleaved coded multiple beamforming," *IEEE Trans. Commun.*, vol. 55, no. 9, pp. 1802–1810, Sep. 2007.
- [2] B. Li, H. J. Park, and E. Ayanoglu, "Constellation precoded multiple beamforming," *IEEE Trans. Commun.*, vol. 59, no. 5, pp. 1275–1286, May 2011.
- [3] B. Li and E. Ayanoglu, "Diversity analysis of bit-interleaved coded multiple beamforming with orthogonal frequency division multiplexing," *IEEE Trans. Commun.*, vol. 61, no. 9, pp. 3794–3805, Sep. 2013.
- [4] Y. Xin, Z. Wang, and G. Giannakis, "Space-time diversity systems based on linear constellation precoding," *IEEE Trans. Wireless Commun.*, vol. 2, no. 2, pp. 294–309, Mar. 2003.
- [5] V. Tarokh, N. Seshadri, and A. Calderbank, "Space-time codes for high data rate communications: Performance criterion and code construction," *IEEE Trans. Inf. Theory*, vol. 44, no. 2, pp. 744–765, Mar. 1998.