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THE DOMATIC NUMBER OF CLEBSCH GRAPH

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Abstract— The Domatic number is the maximum size of domatic partition, that is one can partition the vertex set of G into atleast two disjoint dominating sets. The maximum number of dominating sets into which the vertex set of a graph G can be partitioned. In this paper, we show that the domatic number of Clebsch graph is greater than 4.

Keywords− Domination number, Domatic number, Clebsch graph

I. INTRODUCTION

A Dominating set of graph G is a subset S of the vertex set V(G),such that every vertex of G is either in S or has a neighbor in S. It is well known that the complement of a dominating set of minimum cardinality of a graph G without Isolated vertices is also a dominating set. Hence one can partition the vertex set G into atleast two disjoint dominating sets. The maximum number of dominating sets into which the vertex set of a graph G can be partitioned is called the domatic number of G, and denoted by dom(G). This graph invariant was introduced by Cockayne and Hedetniemi. The word domination is an amalgamation of the word 'domination' and 'chromatic' refers to an anology between the chromatic number (partitioning of the vertex set into independent sets) and the Domatic number (partitioning into dominating sets).For a survey of results on the domatic number of graphs we refer the reader . It was first observed by Cockayne and Hedetniemi that for every graph without isolated vertices $2 \leq \text{dom}(G) \leq \delta+1$, where δ is the minimum degree of G.

Intuitively, it seems reasonable to expect that a graph with large minimum degree will have large domatic number. Zelinka showed that this is not necessarily the case. He gave examples for graphs of arbitrarily large minimum degree with domatic number 2. In this paper we study the domatic number of Clebsch Graph we focus the two aspects of the domatic number of Clebsch graph: properties and construction of Clebsch graph and domatic number of Clebsch graph. In the first part of the paper we show that the properties and construction of Clebsch graph and in the second part we proved that the domatic number of Clebsch graph is greater than 4.

II. NOTATIONS

1. V (G) – vertex set
2. E (G) – edge set
3. dom (G) – domatic number

III. DOMATIC NUMBER

A. Definition

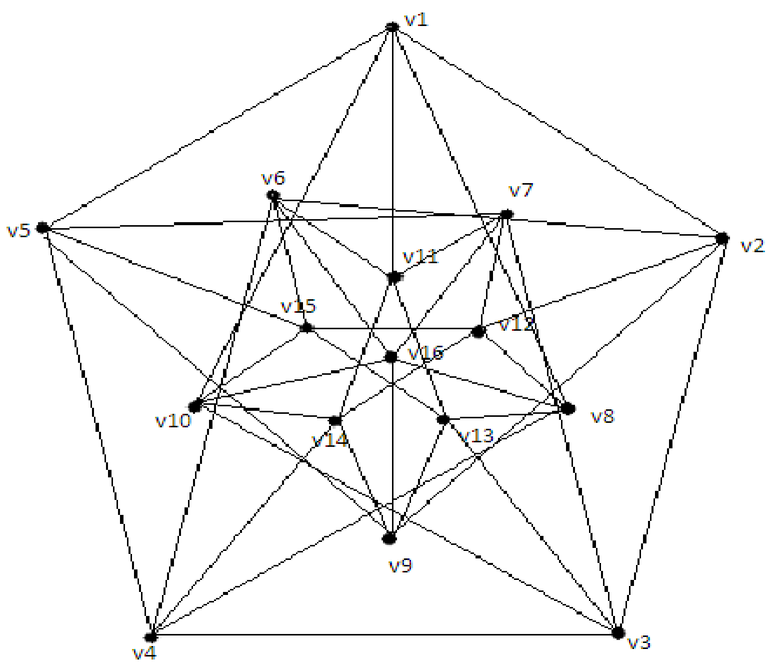
A set S of vertices in a graph G= (V, E) is a dominating set of G if every vertex in V-S is adjacent to some vertex in S. The domination number γ (G) = γ of G is the minimum cardinality of a dominating sets.

B.Definition

The **domatic number** is the maximum size of a domatic partition, that is, the maximum number of disjoint dominating sets. One can partition the vertex set G into atleast two disjoint dominating sets. The maximum number of dominating sets into which the vertex set of a graph G can be partitioned is called the domatic number of G, and denoted by dom(G).

C.Definition

The **Clebsch graph** is a strongly regular Quintic graph on 16 vertices and 40 edges with parameters (γ, k, λ, μ) = (16, 5, 0 and 2). It is also known as the **Greenwood-Gleason Graph**



IV. CONSTRUCTION OF CLEBSCH GRAPH:

1. Let the total number of vertices be v(x) = zỤ V_1Ụ V_2
2. The vertices in are (1, 2, 3, 4, 5). They form an independent set.
3. The vertices in are denoted by 2-sized subsets of the set (1, 2, 3, 4, 5). The

   two subsets are adjacent if they are disjoint (i.e. if they have no co-ordinate in common).
4. The vertices in induce a copy of the Petersen graph.
5. The vertex z is adjacent to all vertices in V1.

6. The vertices in V1 labeled i:(i=1,2,3,4,5) is adjacent to those vertices in V2

Whose label contains as one of the co-ordinates.

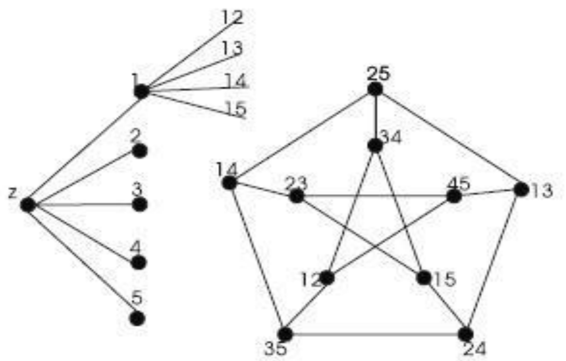
Vertex 1 is adjacent to 12,13,14,15

Vertex 2 is adjacent to 12,23,24,25

Vertex 3 is adjacent to 13,23,34,35

Vertex 4 is adjacent to 14,24,34,45

Vertex 5 is adjacent to 15,25,35,45



V. PROPERTIES OF A CLEBSCH GRAPH:

1. The chromatic number of the Clebsch graph is 4.
2. It is the local graph of the Schlafli graph. The sub graph of the Schlafli graph on the set of non-neighbors of a vertex is the Clebsch graph
3. If a loop is added to each vertex of the Clebsch Graph, the resulting adjacency matrix is equivalent to a 2-(16, 6, 2) block design.
4. The complement of the Clebsch graph is a strongly regular graph. It has least Eigen value -2. It can be represented in.
5. The subgraph on the non-neighbors of a point in the Clebsch Graph is the Petersen graph.
6. The order of the largest independent set in a Clebsch graph is 5

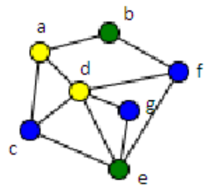
7. Clebsch Graph is triangles free Graph.
8. Every triangle-free planar graph has a homomorphism to Clebsch graph

Theorem:

For any graph, dom(G) > γ(G)

Proof:

let V(G) be the vertices of the graph such that V(G)={ a,b,c,d,e,f,g}
Let dom(G) be the domatic number of graph G and γ(G) be dominating number of graph G.



Then {a,d} are the vertices with minimum cardinality which dominates the whole graph G.
By the definition of domatic number
Hence one can partition the vertex set G into atleast two disjoint dominating sets. The maximum number of dominating sets into which the vertex set of a graph G can be partitioned.
Therefore the dominating sets are
V_1= {a,d}
V_2= {e,b}
V_3={c,g,f}

V_1 consists of yellow vertices
V_2 consists of green vertices
V_3 consists of blue vertices
Thus V_3 dominates maximum number of vertices.
Therefore V_3 has 3 vertices and it is easy to see that the domatic number is atleast 3.

$\therefore$ dom(G) > γ(G)

Theorem:

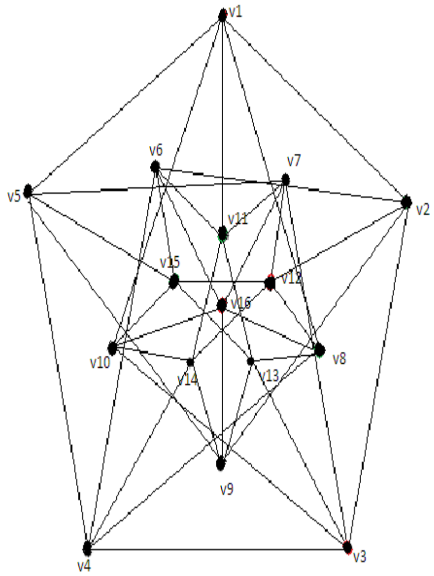
Let G be a Clebsch Graph, then
dom (G) > 4.

Proof:

let V(G) be the vertices of the graph such that V(G)={v_1,v_2,v_3,v_4…v_{16}} by the above theorem we know that "For any Graph dom(G) > γ(G)" by the definition of clebsch Graph "The Clebsch graph is a strongly regular Quintic graph on 16 vertices and 40 edges with parameters (γ, k, λ, μ) = (16, 5, 0, 2). It is also known as the Greenwood-Gleason Graph" i.e. Quintic graph is a graph which is 5-regular
Let S= {v_1, v12, v16, v3} are the vertices with minimum cardinality which dominates the whole graph G.

Then the domination number of Clebsch graph is 4, and then takes another set of vertices which dominates the graph G which is disjoint from S.

Continuing this process until we get disjoint set of vertices, by the definition of Domatic number, one can partition the vertex set G into atleast two disjoint dominating sets. The maximum number of dominating sets into which the vertex set of a graph G can be partitioned is called the domatic number of G, and denoted by



dom(G).

Thus S = {v_1, v12, v16, v3}, γ (G) = 4 and
then another disjoint maximum vertices are {v_5,v_2,v_{10},v_{11},v_8}
∴the maximum number of vertices which dominates the graph is {v_5, v2, v10, v11, v8}
Then the domatic number is 5.

$$\therefore dom(G) > \gamma(G).$$

VI. CONCLUSION

In this paper we have found the domatic number of Clebsch graph is greater than the domination number.

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