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International Journal of Innovative Research in Technology, Science & Engineering (IJIRTSE)
ISSN: 2395-5619, Volume – 1, Issue – 5. July 2015

POPULATION DYNAMICS IN KARNATAKA BY USING VON BERTALANFFY GROWTH MODEL

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***Abstract* - In this article, the paper deals with the application of Logistic equation model in Von Bertalanffy model of the population growth for Karnataka population using the data from 2000 to 2014. The data were collected from the census of Karnataka. Then the Von Bertalanffy model is the tool for the population growth of Karnataka during the year from 2001 to 2030. The parameters a, b, k and L_∞ are used in** Von **Bertalanffy model.**

Keywords: Von Bertalanffy model, population size, population growth, vital coefficient.

I. INTRODUCTION

Population projection has become one of the most important problems in India. In India, the state Karnataka is comprised of 30 districts, grouped into four divisions. Karnataka is located in southwestern part of India. It is the nith largest state in India interms of population. Mathematical modeling is a board inter disciplinary science that was mathematical and computational techniques to model and elucidate the phenomena arising in life science mathematical models can take many forms, including but not limited to dynamical system, statistical models and differential equations. In this paper we model the population growth of Karnataka by using Logistic growth model and Von Bertalanffy growth model.

II. MODELS

- The Logistic Growth model (LGM).
- The Von Bertalanffy model (VBM).

III. DEVELOPMENT OF MODELS

A. THE LOGISTIC GROWTH MODE

The logistic growth equation was introduced in 1838 by Pierre- Francois Verhulst. He proposed the logistic model in which the rate of population growth is the maximum growth rate r, time a modulating factor which is the difference between an optimal size m and the current population size.

The logistic equation involves two positive parameters. First parameter is r and the second parameter is k is called the carrying capacity.

The logistic differential equations is,

$$- = \left(- - \right) \qquad (1)$$

$$\frac{1}{\left(- - \right)} = \qquad (2)$$

Integrating,

$$\int \frac{1}{\left(- - \right)} = \int$$

$$\int \frac{}{\left(- \right)} = \int$$

$$\int \frac{1}{} + \int \frac{}{-} = \int \qquad (3)$$

$$\ln| \,| - \ln| - | = + \qquad (4)$$

Thus,

$$\left| \frac{}{-} \right| = +$$

$$\left| \frac{-}{} \right| = - -$$

Exponentiating,

$$\left| \frac{-}{} \right| = - - \qquad (5)$$

$$= - -$$

$$\frac{-}{} = \pm - -$$

Here $= -$ and $\frac{-}{} = -$,

$$= \frac{}{+ -}, \text{ where } = \frac{-}{} \qquad (6)$$

So the general solution of logistic differential equation is,

$$(\,) = \frac{}{+ -} \text{ for } A \in R. \qquad (7)$$

B. THE VON BERTALANFFY MODEL

The Von Bertalanffy model has been introduced by Ludwig Von Bertalanffy in 1934. Then Von Bertalanffy limited growth model is widely used in fish industry. The model assumes that there is a maximum population growth L_∞, that the population will attain under optimal conditions and that the rate of change in human population, $\frac{}{}$ is proportional to the different between the maximum population growth and the current population, that is,

$$- = \left({}_{\infty} - \right) \qquad (8)$$

Solving this differential equation, then the general form of the Von Bertalanffy limited growth model is

$$- = (_{\infty} -)$$

$$\frac{}{_{\infty} -} =$$

Integrating,

$$\int \frac{}{_{\infty} -} = \int \quad (9)$$

$$-\ln|_{\infty} - | = \quad +$$

$$\ln|_{\infty} - | = - \quad - \quad (10)$$

$$_{\infty} - \quad = \quad ^{- -}$$

$$= \quad _{\infty} - \quad ^{-(+)}$$

$$= \quad _{\infty} - \quad ^{-(+)} \quad (11)$$

The solution to the differential equation is commonly written as

$$() = \quad _{\infty}\left[1 - \quad ^{-(- _{0})}\right]$$

The constant has the property that $(_{0}) = 0$

Solution may be converted from $= \quad _{\infty} - \quad ^{-(+)}$ to

$$() = \quad _{\infty}\left[1 - \quad ^{-(- _{0})}\right] \quad (12)$$

by letting $= \frac{-\ln(_{\infty})}{} - \quad _{0}$.

Here, $= - \quad ()$,

$_{\infty} = - -$ and $_{0} = \frac{}{1 -}$

L(t) is the population growth at t

L_{∞} is the maximum population growth

k is growth coefficient.

IV. ANALYSIS

Table (1) Actual value of population in Karnataka.

YEAR	ACTUAL POLPULATION

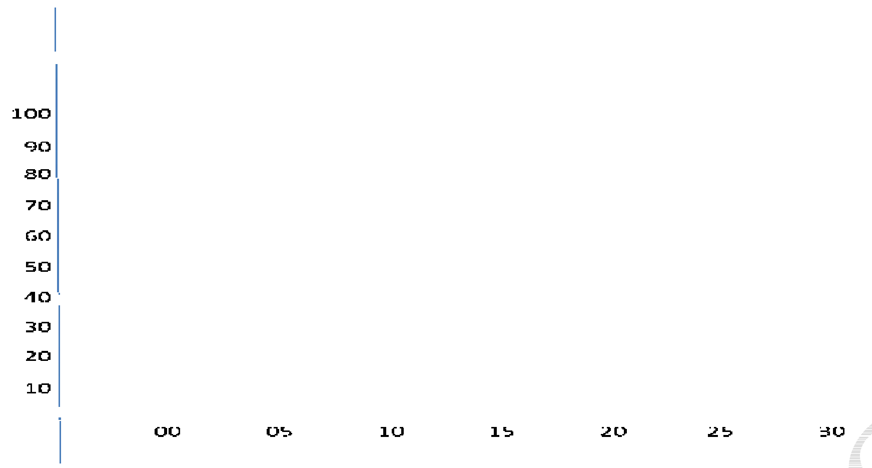
2000	51733958
2001	52850562
2002	53554272
2003	54246861
2004	54926212
2005	55599230
2006	56258340
2007	57550470
2008	58181000
2009	59419681
2010	60624105
2011	61095297
2012	62090456
2013	63065276
2014	64055400

Table (2) Predicted value of population in Karnataka.

YEAR	PREDICTED POPULATION
2000	51751216
2005	55600930
2010	60624607
2015	65076341
2020	69865267
2025	76529702
2030	79852990

Figure (1)

The following graph gives the predicted values of population growth of Karnataka.



Note: The row represents the number of populations in Karnataka.
The column represents the years in two thousand's.

V.DISCUSSION

Using the actual population values, their corresponding years from table (1), we found that the values a and t are 0.0137 and 2030 respectively. The population growth rate of Karnataka according to the information from table (1) was calculated approximately 1.3% in 2001, 1.2% in 2002, 2003, 2004 and 2005, 2.2% in 2006, 1.1% in 2007, 2.1% in 2008 and 2009, 0.8% in 2010, 1.6% in 2011, 2012, 2013 and 2014. Thus the population growth rate of Karnataka is approximately 1.4% per annum. We found the parameters a, b, k and L_∞ are 0.0137, 1.70356×10^{-10}, 22.49313 and 80419667 respectively. Table (2) refers to the predicted population growth value of the year's 2000, 2005, 2010, 2015, 2020, 2025 and 2030, then the corresponding values are illustrated in figure (1).

VI. CONCLUSION

In conclusion, we found that the predicted carrying capacity for the population of Karnataka is 80419667 in 2030. Population size of any country depends also on the vital coefficients. In the case of Karnataka, we found out that the vital coefficients a and b are 0.0137 and 1.70356×10^{-10} respectively. Thus the population growth rate of Karnataka according to Von Bertalanffy model is 1.4% per annum and the value of k=22.49313. Technological developments, pollution and social trends have significant influence on the vital coefficients a and b, therefore, they must re-evaluated every few years to enhance the determination of variation in the population growth rate the government should take step to control the Population growth because it is a major barrier to the development of an Indian economic.

VII. ACKNOWLEDGEMENT

The author would like to sincerest thanks to all the professor's in our department and anonymous reviewer for their constructive suggestions which have considerably contributed to the readability of this paper.

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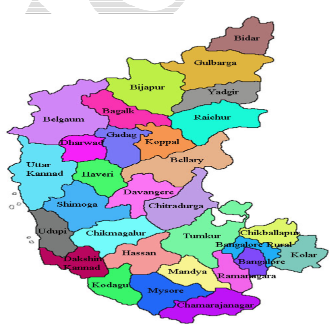
Appendix

Map of Karnataka **Figure (2)**



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