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On 2-rainbow domination number in Harary graph

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Abstract: **The k –rainbow domination is a variant of the classical domination problem in graphs and is defined as follows: Given an undirected graph G= (V, E) and a set of k colors numbered 1,2,….,k, we assign an arbitrary subset of these colors to each vertex of G. If a vertex is assigned the empty set, then the union of color sets of its neighbors must be k-colors. This minimum sum of numbers of assigned colors over all vertices of G is called the K-rainbow domination number of G. In this paper, we find the 2-rainbow domination number of the even regular Harary graphs $,\ ,\ \geq\ +$ and an odd regular Harary graphs $+\ ,\ ,\ \geq\ +$.**

Keywords: Domination number, 2-rainbow domination number, Harary graphs, 2-rainbow domination on Harary graph.

I.INTRODUCTION

A subset S of the vertex set V (G) of a graph G is called a **dominating set** if every vertex in V(G) $-$ S is adjacent to a vertex in S. The **domination number** γ (G) is the minimum cardinality of a dominating set of G.

Rainbow domination was first introduced by Brešar, Henning and Rall in 2005. Let G be a graph and v $\in$ V (G). The open neighborhood of v is the set N (v) = {u $\in$ V (G) | uv $\in$ E (G)} and its closed neighborhood is the set N[v] = N(v) $\cup$ {v}. Let f: V(G) $\rightarrow$ □{1,2,…,k} be a function that assigns to each vertex of G a set of colors chosen from the power set of {1,2,….,k}. If for each vertex $\in ()$ with $() = \emptyset$, $\bigcup_{\in ()} ()$ = {1, 2,…,k}, then the function f is called **a k-rainbow dominating function (KRDF)** of G. The weight of the function f, denoted by w (f) is defined as w (f) = $\sum_{\in ()} |\ ()|$ .The minimum weight of a KRDF is called the **K-rainbow domination number** of G and is denoted by (G).

A 2-rainbow domination function of a graph G is a particular case of KRDF i.e., when k=2. Let us defined as f: V (G) $\rightarrow$ □{1,2} such that for each vertex v $\in$ V(G) with f(v)= $\emptyset$, we have $\bigcup_{\in ()} ()$ = {1,2}. Such a function f is called a **2-rainbow domination function (2RDF)** and minimum weight of such function is called the **2-rainbow domination number** of G and is denoted by γ_{r2}(G).

For example, a 2-rainbow domination ofpath P_5 is shown in Figure 1. We assign a color set {1} to V_1, V_5 and assign a color set {2} to V_3. When f(V_2) = ϕ and f(V_4) = ϕ then $\bigcup_{u\in N(V_2)} f(u)$ = {1,2} and $\bigcup_{u\in N(V_4)} f(u)$ = {1,2}. In fact γ_{r2}(P_5) = 3.

$V_1 V_2 V_3 V_4 V_5$

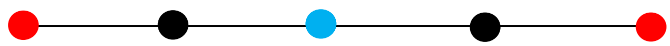


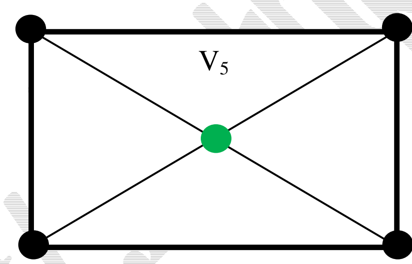
Figure 1 (P_5)

In Figure 1 Path P_5 depicted with a 2-rainbow domination. The vertex with color sets {1} and {2} are filled with colors red and blue respectively.

For another example, 2-rainbow domination of graph G with 5 vertices is shown in Figure 2. We assign a color set {1,2} to V_5. For each vertex v in {V_1, V2, V3, V4}, f (v) = ϕ and $\bigcup_{u\in N(v)} f(u)$ = {1, 2}. Then γ_{r2}(G) = 2.

V_1 V_2

VV



$V_4 V_3$

Figure 2– **(G)**

In Figure 2 the graph G with 5 vertices is shown with 2 rainbow domination. The vertex with color set {1, 2} is filled with green vertex.

In this paper we consider 2-rainbow domination number for the Harary graphs

(i) $H_{2r,n}$, n$\geq$ 2k+1 and (ii) $H_{2r+1,n}$, n$\geq$ 2k+2

II. DEFINITION OF HARARY GRAPH

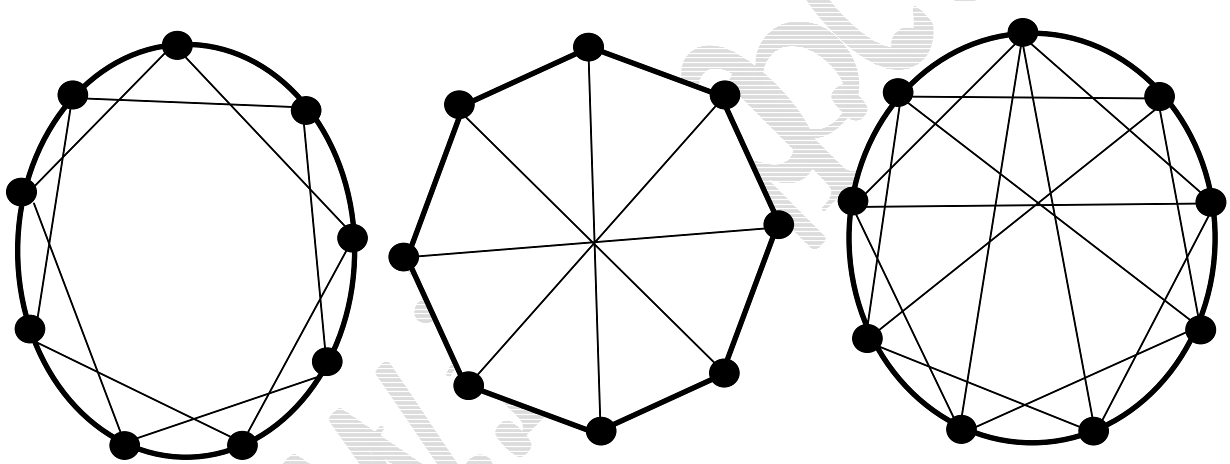
The Harary graph $H_{k,n}$ is defined as follows:

Case 1: When k is even and n is odd. Let k=2r, then $H_{2r,n}$ is constructed as follows : It has vertices 0,1,…..,n$-$ 1 and two vertices i and j are joined if i$-$ r $\leq$ j$\leq$ i+r (where addition is taken modulo n).

Case 2: When k is odd and n is even. Let k=2r+1, then 2 +1, is constructed by drawing first 2 , and then adding edges joining vertex i to vertex i+(n/2) for 1 $\leq$ i $\leq$ n/2.

Case 3: When k is odd and n is odd. Let k=2r=1,then 2 +1, is constructed by first drawing 2 , and then adding edges joining vertex 0 to vertices (n$-$ 1)/2 and vertex i to vertex i+(n+1)/2 for 1 $\leq$ i $\leq$ (n$-$ 1)/2.

The graphs 4,9 , 3,8 , 5,9 are shown as examples of these cases in Figure 3.



4,9 3,8 5,9

Figure – 3

III. THE 2-RAINBOW DOMINATION NUMBER OF 2 , , $\geq 2 +1$

The following Proposition gives a domination number of Harary graph 2 , , n$\geq$ 2k+1.

Proposition 3.1. For $\geq 2 +1$, γ (2 ,) $\leq \lceil 2 +1 \rceil$ where k=1, 2, …

Proof.In a Harary graph 2 , , $\geq 2 +1$ a vertex say v is adjacent to 2k other vertices. Thus there exists a set of 2k+1 vertices among which one vertex is adjacent to the remaining 2k vertices in Harary graph. We choose that vertex as an element of the dominating set. Thus we can partition the vertex set of 2 , , $\geq 2 +1$ into $\lceil 2 +1 \rceil$

subsets. We form a set S such that | S | = $\lceil 2 \ \ +1 \rceil$ and exactly one element of the above $\lceil 2 \ \ +1 \rceil$ subsets in an element of S. Thus γ (2 ,) $\leq$ $\lceil 2 \ \ +1 \rceil$.

The following corollary and Theorem gives the 2-rainbow domination number for Harary graph 2 , .

Corollary 3.2.For $\geq 2 \ \ +1$, 2 (2 ,) $\leq 2 \lceil 2 \ \ +1 \rceil$ where k=1, 2,… .

Proof.Let 2 , , $\geq 2 \ \ +1$ be a Harary graph. Let S be the dominating set of 2 , .Define f:V ((2 ,)) → {1, 2} such that

$$(\ ) = \begin{cases} \{1,2\} & \in \\ \emptyset, & \notin \end{cases}$$ Where i = 1, .2,…., n.

$\bigcup_{\in (\ )} (\ )$ = {1,2} Where $\notin$ S and i = 1, 2,…, n.

Clearly ,f is a 2-rainbow domination function and since each vertex in the dominating set is assigned with a set of two colors and there are a total of $\lceil 2 \ \ +1 \rceil$ vertices in the dominating set of 2 , , $\geq 2 \ \ +1$, we deduce that 2 (2 ,) $\leq$ 2$\lceil 2 \ \ +1 \rceil$, $\geq 2 \ \ +1$ and k=1,2,…. .

Theorem 3.3.For $\geq 2 \ \ +1$, 2 (2 ,) $\geq \lceil \ \ +1 \rceil$ where k=1, 2,…

Proof. Let 2 , be a graph with |V(2 ,) | = n.Let f : V(2 ,) → {1,2} be a 2RDF of 2 , of minimum weight. Let S = { x$\in$ V(2 ,) : f(x) = $\emptyset$ }.Then for every u $\in$ (V(2 ,) S), we have $|(( ))| \geq$ 2. By summing up these inequalities for all vertices of (V ((2 ,))| S) we get,

$$\sum_{\in ((2 ,) -)} |(( ))| \geq 2(| V(2 ,)| - |S|).$$

That is, $\sum_{\in ((2 ,) -)} |(( ))| \geq$ 2(| V(2 ,)| − 2 (2 ,)) where every vertex of S is adjacent to 2k vertices of (V(2 ,) S) . So each weight is counted exactly 2k times on the left hand-side of the above inequality. Thus,

$$(2k) \ 2 (2 ,) \geq 2 (|V(2 ,)| - 2 (2 ,)),$$

$$(2k+2) \ 2 (2 ,) \geq 2 | V(2 ,)|,$$

$$2(k+1) \ 2(2 ,) \geq 2n,$$

$$2(2 ,) \geq \frac{}{+1}.$$

By the definition of 2-rainbow domination number, 2 is an integer.

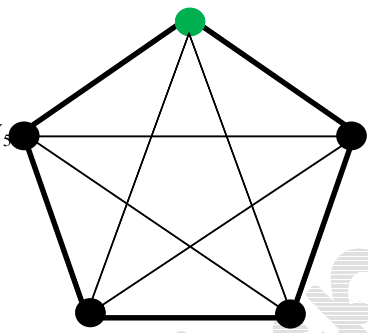
Hence $_2(\ _{2,}\ ) \geq \lceil\ +1 \rceil$ where $\geq 2\ +1$ and k = 1,2,…..

Example: 2-rainbow domination number for $_{4,5}$

Here we take k=2 and $\geq\ +1$ i.e. ≥ 5 then $_{2,}$ becomes $_{4,5}$ and the 2-rainbow domination number of $_{4,5}$ is given in figure 4.Here γ($_{4,5}$)=1. We assign a color set {1,2} to V_1. For each vertex v in {V_2,V_3,V_4,V_5}, f(v) = $\emptyset$ and $\bigcup_{\in ()} ()$ = {1,2}. Then $_2(\ _{4,5})$ = 2.

V_1

V_2 V_5



V_3 V_4

Figure 4 - ($_{4,5}$)

In Figure 4 the Harary graph of $_{4,5}$ is depicted with 2-rainbow domination. The vertex with color set [1,2} is filled with green vertex.

IV. THE 2-RAINBOW DOMINATION NUMBER OF $_{2+1,}$, $\geq 2\ +2$

The following proposition gives the domination number of Harary graph $_{2+1,}$, n$\geq$ 2k+2.

Proposition 4.1. For n$\geq$ 2k+2, γ($_{2+1,}$) $\leq \lceil 2\ +2 \rceil$ where k=1, 2,… .

Proof.In a Harary graph $_{2+1,}$, $\geq 2\ +2$ a vertex says v is adjacent to 2k+1 other vertices. Thus there exists a set of 2k+2 vertices among which one vertex is adjacent to the remaining 2k+1 vertices in Harary graph. We choose that vertex as an element of the dominating set. Thus we can partition the vertex set of $_{2+1,}$, $\geq 2\ +2$ into

$\left\lceil \overline{2\ \ +2} \right\rceil$ subsets. We form a set S such that | S | = $\left\lceil \overline{2\ \ +2} \right\rceil$ and exactly one element of the above $\left\lceil \overline{2\ \ +2} \right\rceil$ subsets in an element of S. Thus γ ($_{2\ +1,}$) $\leq \left\lceil \overline{2\ \ +2} \right\rceil$ where k=1, 2 ,…. .

The following Corollary and Theorem gives the 2-rainbow domination number for Harary graph $_{2\ +1,}$, n$\geq$ 2k+2.

Corollary 4.2.For $\geq 2\ \ +2$, $_{2}(\ _{2\ +1,}\ ) \leq 2 \left\lceil \overline{2\ \ +2} \right\rceil$ where k=1,2,… .

Proof.Let $_{2\ +1,}$, $\geq 2\ \ +2$ be a Harary graph. Let S be dominating set of $_{2\ +1,}$. Define f: V ($_{2\ +1,}$) → {1, 2} such that

$$(\ ) = \begin{cases} \{1,2\} & \in \\ \emptyset, & \notin \end{cases}$$ Where i = 1, 2,…, n.

$$\bigcup_{\in(\ )} (\ ) = \{1,2\}$$ Where $\notin$ S and i = 1, 2,…, n.

Clearly, f is a 2-rainbow domination function and since each vertex in the dominating set is assigned with a set of two colors and there are a total of $\left\lceil \overline{2\ \ +2} \right\rceil$ vertices in the dominating set of $_{2\ +1,}$, $\geq 2\ \ +2$, we deduce that $_{2}(\ _{2\ +1,}\ ) \leq 2 \left\lceil \overline{2\ \ +2} \right\rceil$ where k=1,2,… and $\geq 2\ \ +2$.

Theorem 4.3. For $\geq 2\ \ +2$, $_{2}\ (\ _{2\ +1,}\ ) \geq \left\lceil \frac{2}{2\ \ +3} \right\rceil$ where k=1,2,… .

Proof. Let $_{2\ +1,}$ be a graph with |V ($_{(2\ \ +1,\ )}$) | = n. Let f: V($_{2\ +1,}$) → {1,2} be a 2RDF of $_{2\ +1,}$ of minimum weight. Let S = {x$\in$ V ($_{2\ +1,}$): f(x) = $\emptyset$ }.Then for every u $\in$ (V($_{2\ +1,}$) S), we have $|\ (\ (\ ))| \geq$ 2. By summing up these inequalities for all vertices of (V ($_{(2\ \ +1,\ )}$) S) we get,

$$\sum_{\in((\ _{2\ +1,}\ )-\ )} |\ (\ (\ ))| \geq 2(\ |\ V(\ _{2\ +1,}\ )| - |S|\ ).$$

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That is, $\sum_{\in((z+1,)-)} |(())| \geq$ 2(| V(z +1,)| − z (z +1,)) where every vertex of S is adjacent to 2k+1 vertices of (V(z +1,) S) . So, each weight is counted exactly 2k+1times on the left hand-side of the aboveinequality. Thus,

$$(2k+1)\ z(z+1,) \geq 2\,(|V(z+1,)| - z(z+1,)),$$

(2k+3) z (z +1,) ≥ 2 | V (z +1,)|,

(2k+3) z(z +1,) ≥ 2n,

$$z(z+1,) \geq \frac{2}{2+3}.$$

By the definition of 2-rainbow domination number, z is an integer.

Hence $z(z+1,) \geq \left\lceil \frac{2}{2+3} \right\rceil$ where $\geq 2 + 2$ and k = 1,2,…..

Example: *2-rainbow domination number for* 7,8

Here we take k=3 and then z , becomes 7,8 and the 2-rainbow domination number of 7,8 is given infigure 5.Here γ(7,8)=1. We assign a color set {1,2} to V_1. For each vertex v in {V_2,V_3,V_4,V_5,V_6,V_7,V_8}, f(v) = ∅ and $\bigcup_{\in()} ()$ = {1,2}. Then z (7,8) = 2

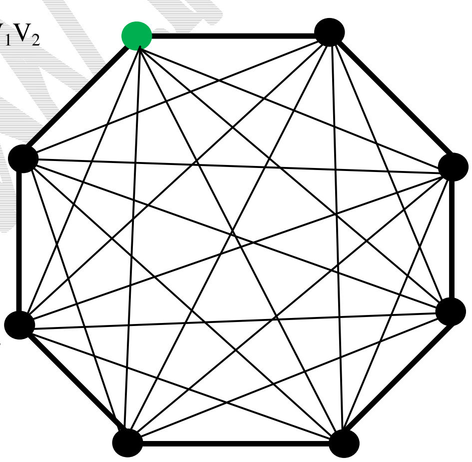

Figure 5 - ( ) 7,8)

In Figure 5 the Harary graph $_{7,8}$ is depicted with 2-rainbow domination. The vertex with the color set {1, 2} is filled with green vertex.

V. APPLICATIONS

There are many applications in 2-rainbow domination such as coding theory, cryptography, networking, computer graphics, image compressing, etc.

VI. CONCLUSION

In this paper we have studied bound on 2-rainbow domination number of even – regular Harary graph $_{2,}$ $\geq 2 + 1$ and odd – regular Harary graph $_{2+1,}$ $\geq 2 + 2$. This work could be further extended to other classes of graphs also.

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